Enrollment No: _______________________Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2022

Subject Name:Real Analysis-I

Subject Code:4SC05REA1 **Branch: B.Sc. (Mathematics)**

Semester: 5 **Date: 27/04/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1	**Attempt the following questions:**	**[14]**
a)	Define: Cauchy Sequence and Monotonic increasing sequence.	(02)
b)	State Raabe's test for series.	(02)
c)	Find the infimum and supremum of $\left\{\frac{1}{n} : n \in N\right\}$.	(02)
d)	Define: Continuity at a point.	(02)
e)	Find the range set of the sequence $\{1 + (-1)^n : n \in N\}$.	(02)
f)	Check the series $\sum_{n=1}^{\infty} \left(\frac{1}{8}\right)^n$ is converges or diverges.	(01)
g)	True/False: Every bounded sequence is convergent.	(01)
h)	Define: Bounded sequence	(01)
i)	True/False: $\sum \frac{1}{n^3}$ is divergent.	(01)

Attempt any four questions from Q-2 to Q-8

Q-2	**Attempt all questions**	**[14]**
a)	State and prove Bolzano Weierstrass theorem for sequences.	(07)
b)	Prove : $\lim_{n\to\infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$ using definition.	(05)
c)	Define: Limit Inferior and Limit Superior of a sequence.	(02)

Q-3	**Attempt all questions**	**[14]**
a)	Using Sandwich theorem, prove that $\lim_{n\to\infty} \left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \dots + \frac{1}{\sqrt{n^2+n}}\right] = 1$.	(05)
b)	Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$ .State the results you used.	(05)
c)	Prove that every open interval contains a rational number.	(04)

Q-4 Attempt all questions **[14]**

a) Test the convergence of the series $1 + x + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \dots, x > 0$ (07)

b) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2(n+1)^2}{n!}$. (04)

c) Prove that $\lim_{n\to\infty} \frac{1+3+5+\dots+(2n-1)}{n^2} = 1$ (03)

Q-5 Attempt all questions **[14]**

a) Find the right hand and lefthand limits of a function defined as follows: (05)

$$f(x) = \begin{cases} \frac{|x-4|}{x-4} & ; \ x \neq 4 \\ 0 & ; \ x = 4 \end{cases}.$$

b) If $\{a_n\}$ is any sequence then prove the followings: (05)

i. $\underline{\lim}(-a_n) = -\overline{\lim}\, a_n$

ii. $\overline{\lim}(-a_n) = -\underline{\lim}(a_n)$

c) Define : Conditionally Convergent Series and Absolutely Convergent Series. (04)

Q-6 Attempt all questions **[14]**

a) Show that the geometric series $1 + r + r^2 + \dots$ is (09)
i) Convergent if $|r| < 1$, ii) Divergent if $r \geq 1$, iii) Finitely oscillating if $r = -1$ and iv) Infinitely oscillating if $r < -1$.

b) Prove that the series $\frac{1}{2} - \frac{1}{2}\cdot\frac{1}{2^2} + \frac{1}{3}\cdot\frac{1}{2^3} - \frac{1}{4}\cdot\frac{1}{2^4} + \dots$ is absolutely convergent. (05)

Q-7 Attempt all questions **[14]**

a) State and prove Cauchy's general principle of convergence for sequence. (07)

b) Show that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\log(n+1)}$ is conditionally convergent. (05)

c) State Leibnitz Test for Alternating Series. (02)

Q-8 Attempt all questions **[14]**

a) State and prove D'Alembert's ratio test. (10)

b) Prove that $\sin x$ is uniformly continuous on $[0, \infty)$. (04)